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Article

New Aspect in order Reduction of Large-scale Uncertain MIMO System using Modified Routh Approximation and Affine Arithmetic

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I N F O

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A B S T R A C T

This paper presents the Modified Routh Approximation (MRA) and Affine Arithmetic (AA) methods for obtaining the Reduced Order Model (ROM) of Higher Order Model (HOM). The reduced order model of a large-scale Uncertain MIMO System is required to simplify analysis of any system. The proposed method is compared with different MIMO uncertain systems in the literature. It is analyze that the proposed method is nearly identical in behavior to that of with original higher order systems. Observed numerical examples are considered to illustrate the effectiveness of the proposed method.

Keywords: Routh Approximation Method, Affine Arithmetic, Model Order Reduction, MIMO Uncertain Systems

Introduction

The study and design of high order system is too difficult and costly in the real time problems. Using mathematical approach the original high order system can be convert into lower order system. Reduced order system is considered to be essential in investigation, synthesis and model of practical systems. So, it is required to achieve a lower order system and, its maintain the features of the original system.

In frequency area different reduction methods are available for reducing the order of high order SISO system into lower order SISO system, and few methods are available in MIMO system and several methods are available for changing from SISO to MIMO system and it is available in literature. Earlier a method for the reduction of MIMO system was given by Bendekas et.al., which is a mixed method of time moment matching and Pade approximates. Taiwo et.al. developed methods based upon Pade approximated generalized time moments. Chen has given a method based on matrix continued fraction technique. The Routh approximation method given by Hutton was extended to MIMO systems by Sinha .

The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system. Numerical methods are available in the literature for order reduction of large-scale system. Some properties are explained briefly by using Aggregation method. An algorithm was introduced for reduced order model using Pade approximation[1-3], Routh approximation [4] has done by using α and β table and Routh stability for model reduction[3-4] is considered as a Stability based model reduction, moment matching technique[5], Optimal hankel norm approximation. A Pade approximation method has various advantages like computational simplicity. There is a serious disadvantage like unstable reduced model, if original system is stable. To overcome this problem by mixed methods[6] are introduced. Many scholars has concerned the interest on interval system for stability study and the transient analysis. Routh approximation has been proposed [9] for reduction of continuous interval system and γ-δ Routh approximation has been proposed for model order reduction of interval system, to reduce the difficulty of calculation.

In this paper, the Routh Approximation method and Affine Arithmetic are explored for getting the reduced order model of a higher order model. The reduced order modelling of a large system is necessary to ease the analysis of the system. The approach is examined and compared to Multi-Input Multi-Output (MIMO) systems. The response comparison is considered in terms of step response parameters and graphical comparisons. It is reported that the reduced order model using proposed Routh Approximation (RA) method is almost similar in behaviour to that of with original systems

This paper is organized as follows: Reduction procedure is given in Section II. Numerical examples and comparison of proposed method with other well-known techniques is shown in Section III and conclusion and references in Section IV.

Reduction Procedure

In this section, the proposed method for reducing higher order is interpreted. The reduced order coefficients are gathered by applying Modified Routh Approximation Method[1], without considering time instants of the original system. In this proposed method both γ and δ table is calculated from the below equations.

Let the original high order q-input, p-output MIMO interval system be given in transfer matrix form as:

$$G_n(s)=\frac{[N_{i,j}(s)]}{D_n(s)};\text{with } i=1,2,3,\ldots,p;\ j=1,2,3,\ldots,q \text{ and } (m\le n)$$

$$=\begin{bmatrix} N_{11}(s) & N_{12}(s) & \ldots & N_{1q}(s)\\ N_{21}(s) & N_{22}(s) & \ldots & N_{2q}(s)\\ \ldots & \ldots & \ldots & \ldots\\ N_{p1}(s) & \ldots & \ldots & N_{pq}(s)\end{bmatrix} \quad (1)$$

where the common denominator polynomial is:

$$D_n(s)=[A_n^-,A_n^+]s^n+[A_{n-1}^-,A_{n-1}^+]s^{n-1}+\ldots\ldots\ldots+$$

$$[A_1^-,A_1^+]s+[A_0^-,A_0^+]$$

the scalar numerator polynomials are:

for i = 1, j = 1,

$$N_{11}(s)=[B_m^-,B_m^+]_{11}s^m+[B_{m-1}^-,B_{m-1}^+]_{11}s^{m-1}+\ldots\ldots\ldots+$$

$$[B_1^-,B_1^+]_{11}s+[B_0^-,B_0^+]_{11}$$

for i = 1, j = 2,

$$N_{12}(s)=[B_m^-,B_m^+]_{12}s^m+[B_{m-1}^-,B_{m-1}^+]_{12}s^{m-1}+\ldots..+$$

$$[B_1^-,B_1^+]_{12}s+[B_0^-,B_0^+]_{12}$$

and in general,

$$N_{ij}(s)=[B_m^-,B_m^+]_{ij}s^m+[B_{m-1}^-,B_{m-1}^+]_{ij}s^{m-1}+\ldots\ldots\ldots+$$

$$[B_1^-,B_1^+]_{ij}s+[B_0^-,B_0^+]_{ij}$$

It is proposed to obtain a low order model for the above original high order interval system as:

$$R_r(s)=\frac{[n_{i,j}(s)]}{d_r(s)};\text{with } i=1,2,3,\ldots,p;\ j=1,2,3,\ldots,q.$$

$$=\begin{bmatrix} n_{11}(s) & n_{12}(s) & \ldots & n_{1q}(s)\\ n_{21}(s) & n_{22}(s) & \ldots & n_{2q}(s)\\ \ldots & \ldots & \ldots & \ldots\\ n_{p1}(s) & \ldots & \ldots & n_{pq}(s)\end{bmatrix} \quad (2)$$

where the reduced order common denominator polynomial is

$$d_r(s)=[a_r^-,a_r^+]s^r+\ldots\ldots\ldots+[a_1^-,a_1^+]s+[a_0^-,a_0^+] \text{ with } [a_r^-,a_r^+]=[1,1]$$

and the scalar numerator polynomials are

for i = 1, j = 1,

$$n_1(s)=[b_{r-1}^-,b_{r-1}^+]_1\ s^{r-1}+\ldots\ldots\ldots+[b_1^-,b_1^+]_1\ s+[b_0^-,b_0^+]_1$$

for i = 1, j = 2,

$$n_2(s)=[b_{r-1}^-,b_{r-1}^+]_2\ s^{r-1}+\ldots\ldots\ldots+[b_1^-,b_1^+]_2\ s+[b_0^-,b_0^+]_2$$

and in general

$$n_j(s)=[b_{r-1}^-,b_{r-1}^+]_j\ s^{r-1}+\ldots\ldots\ldots+[b_1^-,b_1^+]_j\ s+[b_0^-,b_0^+]_j$$

Consider the transfer function of higher order in the interval system as follows:

$$G(s)=\frac{[d_1^-,d_1^+]s^{n-1}+[d_2^-,d_2^+]s^{n-2}+\cdots+[d_n^-,d_n^+]}{[c_0^-,c_0^+]s^n+[c_1^-,c_1^+]s^{n-1}+\cdots+[c_n^-,c_n^+]} \quad (3)$$

The r^{th} order reduced model $R_r(s)$ is characterized by

$$R_r(s)=\frac{D_r(s)}{C_r(s)} \quad (4)$$

Where $C_r(s)=s^2C_{r-2}(s)+[\gamma_r^-,\gamma_r^+]C_{r-1}(s)$

$$D_r(s)=[\delta_r^-,\delta_r^+]s^{r-1}+s^2D_{r-2}(s)+[\gamma_r^-,\gamma_r^+]D_{r-1}(s)$$
(5)

$$\text{with}\quad A_{-1}(s)=\frac{1}{s};\ A_0(s)=1 \quad\text{and} \quad (6)$$

$$B_{-1}(s)=0;\ B_0(s)=0$$

The interval γ- parameters be obtained from a Routh like table as follows:

γ-table

$$\begin{aligned}
&[c_n^-,c_n^+][c_{n-2}^-,c_{n-2}^+][c_{n-4}^-,c_{n-4}^+]\\
&=[c_{00}^-,c_{00}^+]=[c_{01}^-,c_{01}^+]=[c_{02}^-,c_{02}^+]\\
&[c_{n-1}^-,c_{n-1}^+][c_{n-3}^-,c_{n-3}^+][c_{n-5}^-,c_{n-5}^+]\\
&=[c_{10}^-,c_{10}^+]=[c_{11}^-,c_{11}^+]=[c_{12}^-,c_{12}^+]\\
&\vdots\\
&[c_{n-1,0}^-,c_{n-1,0}^+]\\
&[c_{n,0}^-,c_{n,,0}^+]
\end{aligned} \quad (7)$$

The interval γ-parameters can be defined as the ratios of the first column elements of the above Routh like table as given below:

$$\gamma_k = \frac{[c_{k-1,0}^-, c_{k-1,0}^+]}{[c_{k,0}^-, c_{k,0}^+]} \, for k = 1, 2, 3, \cdots n \tag{8}$$

The interval δ-parameters can be obtained from a table constituted from the coefficients of the numerator of original high order interval system as follows:

δ - table:

$$\begin{array}{|l} \vdots \\ [d_{30}^-, d_{30}^+][d_{31}^-, d_{31}^+][d_{32}^-, d_{32}^+] \\ \vdots \\ [d_{n-1,0}^-, d_{n-1,0}^+] \\ [d_{n,0}^-, d_{n,0}^+] \end{array} \tag{9}$$

The interval δ-parameters can be defined as the ratios of the first column elements of the δ-table as given below:

$$\delta_k = \frac{[d_{k,0}^-, d_{k,0}^+]}{[c_{k,0}^-, c_{k,0}^+]} \, for k = 1, 2, 3, \cdots \cdots n \tag{10}$$

Modified Routh Approximation of large scale reduced uncertain system using Affine Arithmetic is considered. The presented method gives the reduced order system characteristics as similar as the higher order system characteristics. The step response of original higher order system, interval arithmetic and affine arithmetic for both lower and upper bound intervals are separately represented in the figure a, b, c and d.

Numerical Example

Consider a stable 6th order two input, two output interval system given below in transfer matrix form:

$$G_6(s) = \frac{\begin{bmatrix} [2.0,2.2] & [1.0,1.1] \\ [1.0,1.1] & [1.5,1.6] \end{bmatrix} s^5 + \begin{bmatrix} [70,72] & [38,39.5] \\ [30,30.9] & [42,43.5] \end{bmatrix} s^4 + \begin{bmatrix} [762,784] & [459,472] \\ [331,334] & [601,619] \end{bmatrix} s^3 + \begin{bmatrix} [3610,3718] & [2182,2248] \\ [1650,1690] & [3660,3769] \end{bmatrix} s^2 + \begin{bmatrix} [7890,7931] & [4225,4285] \\ [3785,3811] & [9100,9150] \end{bmatrix} s + \begin{bmatrix} [6000,6180] & [2400,2472] \\ [3000,3090] & [6000,6180] \end{bmatrix}}{[1.0,1.1]s^6 + [41,42]s^5 + [571,576]s^4 + [3491,3525]s^3 + [10060,10160]s^2 + [11215,11362]s + [6000,6180]}$$

For the above original high order internal system, the 2nd order reduced model by the proposed method is obtained as given below:

$$R_2(s) = \frac{\begin{bmatrix} [2.01,2.01] & [1.0,1.0] \\ [1.0,1.0] & [1.48,1.48] \end{bmatrix} s + \begin{bmatrix} [8.8583,9.2147] & [3.5433,3.6859] \\ [4.4292,4.6074] & [8.8583,9.2147] \end{bmatrix}}{[1,1]s^2 + [5.5192,5.6471]s + [8.8583,9.2147]}$$

(Proposed Method) *(Stable)*

The step responses of the original high order system with the 2nd order reduced model of proposed method is compared in fig. a - fig. d.

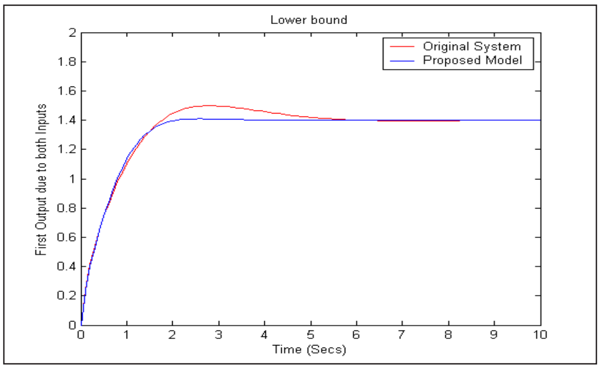

Figure a.Comparison of Step Responses

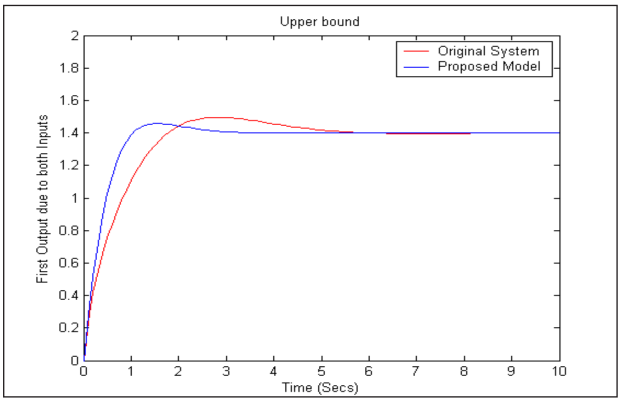

Figure b.Comparison of Step Responses

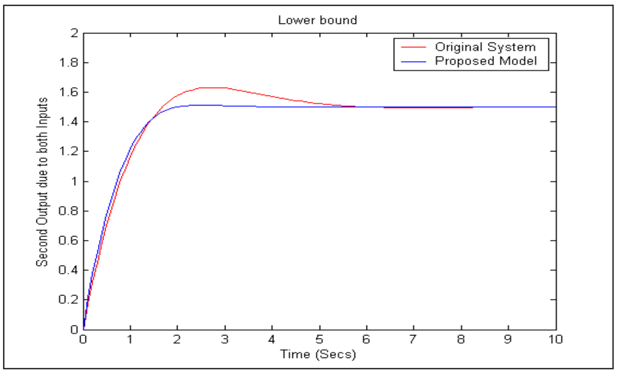

Figure c.Comparison of Step Responses

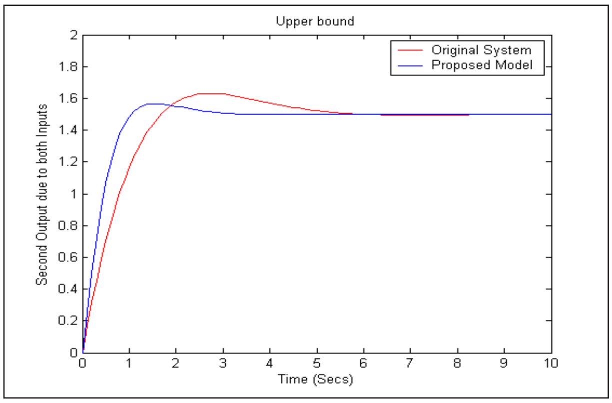

Figure d.Comparison of Step Responses

Case Study

Consider a 3rd order single input, two output stable interval system given by :

$$G(s) = \frac{1}{D(s)} \begin{bmatrix} N_{11}(s) & N_{12}(s) \\ N_{21}(s) & N_{22}(s) \end{bmatrix}$$

Where

$$N_{11}(s) = [11,13]s^2 + [30,32]s + [22,23];$$

$$N_{12}(s) = [3,5]s^2 + [18,19]s + [9,10]$$

$$N_{21}(s) = [1,2]s^2 + [15,18]s + [12,13];$$

$$N_{22}(s) = [7,8]s^2 + [26,29]s + [20,21]$$

And $D(s) = [2,3]s^3 + [34,36]s^2 + [75,78]s + [38,39]$

The 2nd order reduced model obtained by applying the proposed method is given by:

$$R_2'(s) = \frac{1}{d_2(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \quad \textit{(Proposed method)}$$

Where $n_{11}(s) = [1.5083, 1.5083]s + [1.8215, 2.0220]$

$$n_{12}(s) = [1.0750, 1.0750]s + [0.7521, 0.8871]$$

$$n_{21}(s) = [0.6666, 0.6666]s + [1.0089, 1.1480]$$

$$n_{22}(s) = [1.1667, 1.1667]s + [1.676, 1.853] \quad \textit{(Stable)}$$

The second order reduced model obtained by using the method "Stable Multi point Pade Approximation for linear MIMO Structured Uncertain systems" suggested by Ismail is obtained as:

$$R_2'(s) = \frac{1}{d_2(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \quad \textit{(Ismail method)}$$

Where $n_{11}(s) = [38.69, 42.77]s + [22,23]$,

$$n_{12}(s) = [19.9, 22.88]s + [9,10]$$

$$n_{21}(s) = [11.98, 16.06]s + [12,13],$$

$$n_{22}(s) = [31.06, 35.1]s + [20,21]$$

$$d_2(s) = [34,36]s^2 + [71.558, 75.88]s + [38,39]$$

The step response comparison of reduced order models obtained by the proposed method and the method "Stable Multi point Pade Approximation for linear MIMO Structured Uncertain systems" suggested by Ismail with the original system are shown in fig. e- fig .h:

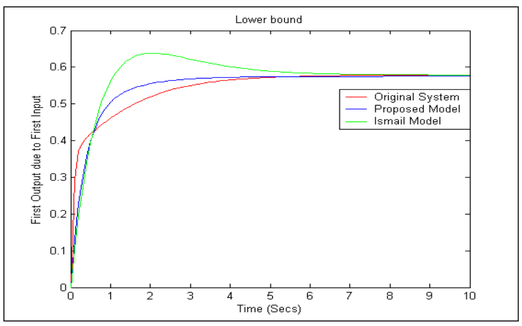

Figure e.Comparison of Step Responses for Lower Limit

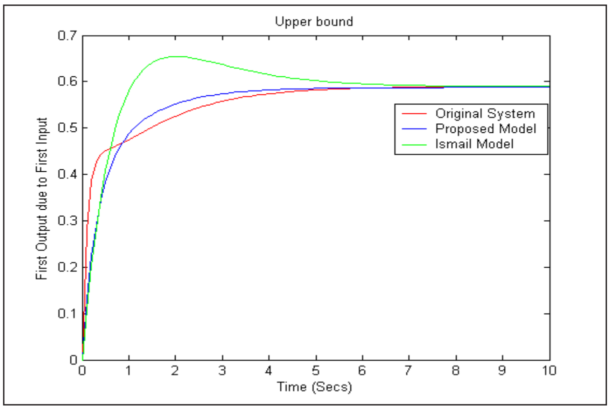

Figure f.Comparison of Step Responses for Upper Limit

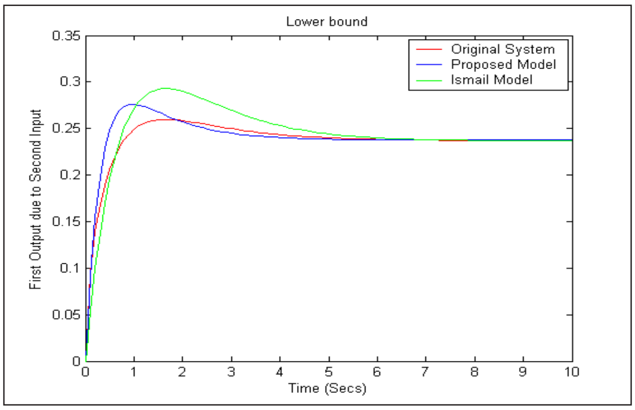

Figure g.Comparison of Step Responses for Lower Limit

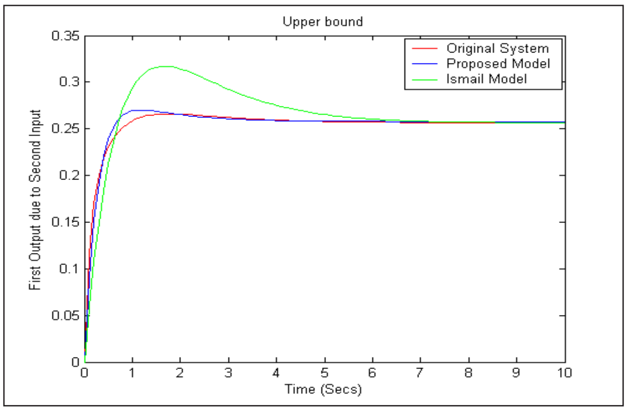

Figure h.Comparison of Step Responses for Upper Limit

Conclusion

It is observed from the literature that, the analysis and design of large-scale Uncertain MIMO System become complex and costly. The proposed method as become a better appliance for analysis and simulation of high-order uncertain systems. It is better to perform from high-order system to lower-order model without allowing the main features of the original system. The developed lower-order system from the original system characterize the performance of the original system is very costly. Model order reduction is applied for simulation of higher-order systems. Affine Arithmetic preserve the correlations between those quantities and its provided better steady intervals than interval arithmetic.

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