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Article

Simple Graphical Methods to Learn Trigonometry

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A B S T R A C T

In this paper, two simple graphical methods to teach and learn trigonometry are presented. The paper is divided into two parts. The first part proposes a graphical approach to memorize the waveforms and the values of the three main trigonometric functions (i.e. tangent, sine, and cosine) when the angle is at 90°. In the second part of the paper, a triangular model has been developed to allow students to memorize trigonometric identities with ease. Five trigonometric identities, namely, the reciprocal, quotient, Pythagorean, product, and derivative identities have been incorporated into the model. By perceiving the model from different lights, the relationships among these five identities could be easily correlated.

Keywords: Cosine, Sine, Tangent, Trigonometry

Introduction

Throughout years of teaching students in the field of Electrical and Electronic Engineering, we found that many of them face difficulty in subjects which involve trigonometric functions. Very often, students find it hard to understand the derivation of formulations in topics such as transmission lines (Cheng, 1989; Matick, 1969) waveguides (Yeap et al., 2017; Yeap et al., 2010a; Yeap et al., 2010b) and microstrip lines (Yeap et al., 2010c; Yassin and Withington, 1995; Wheeler, 1964). This is because the propagation of the time-varying electromagnetic field has to be denoted by the sine and cosine functions. Without the aid of a scientific calculator, students find it hard to visualize the waveforms of these trigonometric functions. Furthermore, the derivation of these functions often involves the manipulation of trigonometric identities. Since it has never been easy to memorize trigonometric identities, students face difficulties in solving or deriving problems which involve the manipulation of trigonometric identities. Due to this reason, students tend to lose interest in these topics. Indeed, a trend of declining interest in engineering has been observed in various countries (Dimopoulos, Katzis, and Hawwash, 2011) and this is partly attributed to the students' difficulty in understanding mathematics and science (Ling, 2016).

To inculcate interest on mathematics among students, teaching approaches which enable them to understand in a relatively easy manner is therefore necessary. In this paper, two simple graphical approaches to learn trigonometry are presented. By sharing these methods to the community, we hope that not only students in the field of Electrical and Electronic Engineering, but students in other fields which require the knowledge of trigonometry, will also benefit from it.

Pictorial Interpretation of The Trigonometric Functions

We discovered that the waveforms, as well as some of the values of the three main trigonometric functions, namely, tan(θ), sin(θ), and cos(θ) can be easily observed from the name itself. As shown in Figure 1, below, the waveforms of these three functions can be easily deduced from the first letter of their trigonometric functions.

If we observe closely the second letter of the functions, we shall find that the letter resembles the value of the functions at angle $\theta = \pi/2$ rad or 90°. Hence, as depicted in Figure 2, the value of tan(π/2), sin(π/2), and cos(π/2) can be deduced from the second letter of the functions.

Moreover, it is also worthwhile noting that, in "cos(θ)", the letter "c" comes before the letter "s". This actually helps us to remember the fact that the cosine function is always leading the sine function (by π/2 rad).\

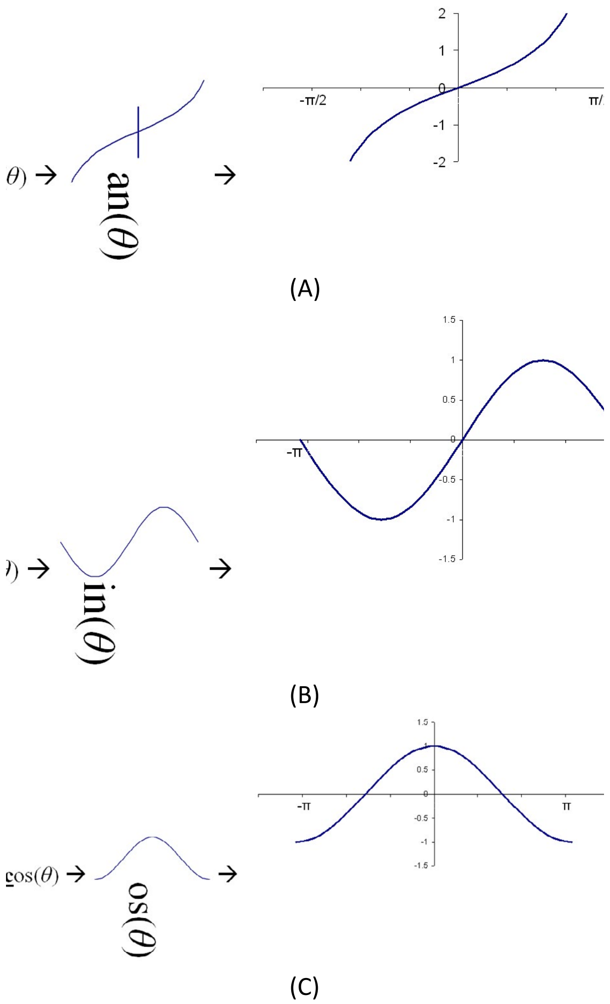

Figure 1.(a) tan(θ) (b) sin(θ) (c) cos(θ)

$$\tan(\pi/2) \rightarrow a \rightarrow \alpha \rightarrow \text{Thus, } \tan(\pi/2) = \alpha$$

(A)

$$\sin(\pi/2) \rightarrow i \rightarrow 1 \rightarrow \text{Thus, } \sin(\pi/2) = 1$$

(B)

$$\cos(\pi/2) \rightarrow 0 \rightarrow 0 \rightarrow \text{Thus, } \cos(\pi/2) = 0$$

(C)

Figure 2.(a) tan(π/2) (b) sin(π/2) (c) cos(π/2)

The Yeap's triangle

In Pierce (2017), the magic hexagon has been applied to memorize some of the trigonometric identities. In order to be able to incorporate a more complete set of trigonometric identities, we have modified the hexagon into a triangle as depicted in Figure 3. For convenience purpose, we shall refer it as the Yeap's triangle.

Here, we shall summarize some of the trigonometric identities incorporated in the Yeap's triangle.

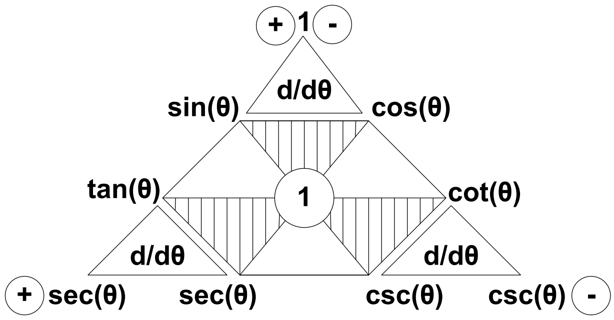

Figure 3.The Yeap's triangle

Reciprocal Identities

The lines in the hexagon which connect the functions to the number one in the circle denote the reciprocal identities of the functions in the opposite direction. Figure 4, depicts graphically an example of one of the reciprocal identities: sin(θ) = 1/csc(θ).

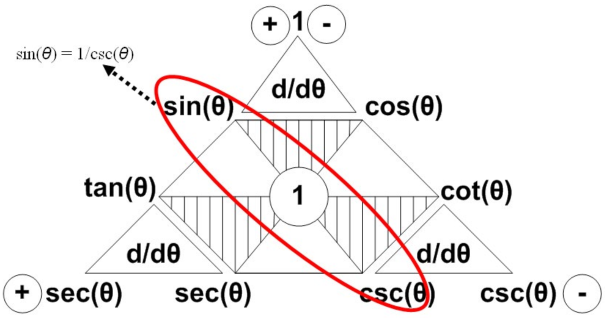

Figure 4.A Graphical Illustration of how Reciprocal Identities can be Obtained from the Triangle Diagram

The list of reciprocal identities is shown below:

tan(θ) = 1/cot(θ)

sin(θ) = 1/csc(θ)

cos(θ) = 1/sec(θ)

cot(θ) = 1/tan(θ)

csc(θ) = 1/sin(θ)

sec(θ) = 1/cos(θ)

Quotient Identities

The division of two successive functions in the hexagon (in clockwise or counter clockwise direction) result in the function located at the vertex before the two. Figure 5, depicts graphically an example of one of the quotient identities: tan(θ) = sin(θ)/cos(θ).

The list of quotient identities is shown below:

tan(θ) = sin(θ)/cos(θ)

sin(θ) = cos(θ)/cot(θ)

cos(θ) = cot(θ)/csc(θ)

cot(θ) = csc(θ)/sec(θ)

csc(θ) = sec(θ)/tan(θ)

sec(θ) = tan(θ)/sin(θ)

cot(θ) = cos(θ)/sin(θ)

csc(θ) = cot(θ)/cos(θ)

sec(θ) = csc(θ)/cot(θ)

tan(θ) = sec(θ)/csc(θ)

sin(θ) = tan(θ)/sec(θ)

cos(θ) = sin(θ)/tan(θ)

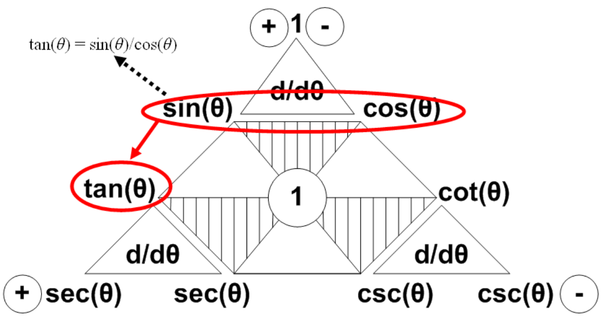

Figure 5.A Ggraphical Illustration of How Quotient Identities can be Obtained from The Triangle Diagram

Pythagorean Identities

The three Pythagorean identities are shown in the three shaded triangles. It is to be noted that the triangles are oriented in such a way that there are two values at the top and one at the bottom. The values are either trigonometric functions or the number one in the circle. The vertical lines which form the shades in the triangles, act as pointers, inferring that the final answer lies at the bottom of the triangles. According to the Pythagorean identities, the sum of the squares of the top two values results in the square of the bottom. Figure 6, depicts graphically an example of one of the quotient identities: $\sin^2(\theta) + \cos^2(\theta) = 1$.

The list of Pythagorean identities is shown below:

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\cot^2(\theta) + 1 = \csc^2(\theta)$

Product Identities

The product of two functions located in between a selected function equals that selected function itself. Figure 7,

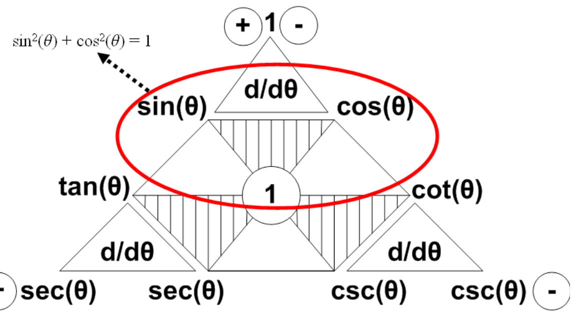

Figure 6.A Graphical Illustration of How Pythagorean Identities can be Obtained from The Triangle Diagram

depicts graphically an example of one of the product identities: tan(θ) = sin(θ)*sec(θ).

The list of product identities is shown below:

tan(θ) = sin(θ)*sec(θ)

sin(θ) = tan(θ)*cos(θ)

cos(θ) = sin(θ)*cot(θ)

cot(θ) = cos(θ)*csc(θ)

csc(θ) = sec(θ)*cot(θ)

sec(θ) = tan(θ)*csc(θ)

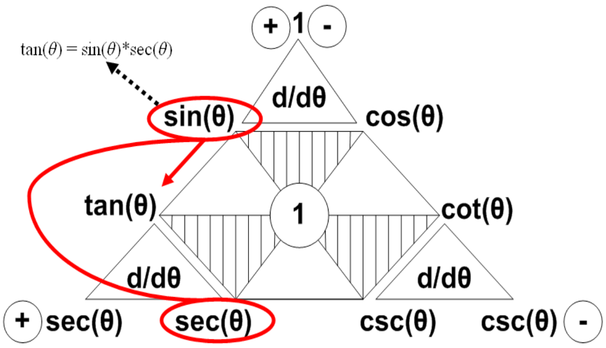

Figure 7.A Graphical Illustration of How Product Identities can be Obtained from The Triangle Diagram

Derivative Identities

The three triangles appended to the corners of the hexagon shows the derivative identities. The solution to the derivative of the function at one corner of the hexagon is given by the product of the values (either a function or a number one) at the remaining two corners of the triangle. It is to be noted that for the derivative of any of the function at the right side of the hexagon (i.e. cos(θ), cot(θ), or csc(θ)), the final answer has to be negative; On the contrary, for the derivative of any of the function at the left side of the hexagon (sin(θ), tan(θ), and sec(θ)), the final answer has to be positive. Figure 8, depicts graphically an example of one of the product identities d/dθ [sin(θ)] = cos(θ)*1.

The list of derivative identities is shown below:

d/dθ[tan(θ)] = sec(θ)*sec(θ)

d/dθ [sin(θ)] = cos(θ)*1

d/dθ [cos(θ)] = −sin(θ)*1

d/dθ [cot(θ)] = −csc(θ)*csc(θ)

d/dθ [csc(θ)] = −csc(θ)*cot(θ)

d/dθ [sec(θ)] = sec(θ)*tan(θ)

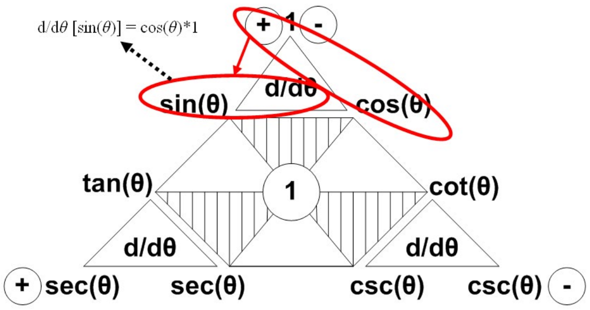

Figure 8.A Graphical Illustration of How Derivative Identities can be Obtained from The Triangle Diagram

Summary

We have presented 2 simple and easy-to-use methods to learn trigonometry. The first method involves a direct pictorial interpretation of the names of the tangent, sine, and cosine functions. By observing the first and second letter of the functions, respectively, we would be able to learn about the waveforms, as well as, the value of the function when θ = 90°. In the second method, we have developed a triangle model which helps to memorize 5 different trigonometric identies, namely, the reciprocal, quotient, Pythagorean, product, and derivative identities. We believe that by employing these two methods in teaching trigonometry the students' interest in solving mathematical problems which involve trigonometric functions could be enhanced.

Biography

Kim Ho Yeap received his B. Eng. (Hons) from Petronas University of Technology in 2004, M.Sc. from National University of Malaysia in 2005 and PhD from Universiti Tunku Abdul Rahman in 2011. He is a senior member of the IEEE, a chartered engineer registered with the UK engineering council and a professional engineer registered with the board of engineers Malaysia. He has published more than 100 scientific articles which include journal and conference papers, book chapters and books. He is currently an Associate Professor in Universiti Tunku Abdul Rahman. He is also the editor-in-chief of i-manager's Journal on Digital Signal Processing.

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